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We discuss the cosmology in four dimensions within a context of brane-world scenario. Such models can predict chaotic inflation with very low reheat temperature depending on the brane tension. We study the perturbative production of gravitinos in the frame-work of brane cosmology and we notice that tuning the brane tension or the five dimensional Planck mass it is possible to obtain a very small abundance for gravitinos. We also study Afleck-Dine mechanism for baryogenesis in our toy model and we notice that it is possible to attain an observed baryon asymmetry which depends on the brane tension but not on the mass parameter responsible for lifting the flat directions in broken supersymmetric theories.

Recently there has been a renewed interest in perceiving the four dimensional world which is in a form of three dimensional hypersurface along with time embedded in a higher dimensional space-time. Such a claim has a pedigree from strongly coupled sector of $E_8 \times E_8$ hetrotic string theory which can be described by a field theory living in a 11 dimensional space-time [1]. The 11 dimensional world comprise of two 10 dimensional hypersurfaces embedded on an orbifold fixed points, where fields are assumed to be confined to the hypersurfaces which are known to be 9 branes in this scenario. After compactifying the 11 dimensional theory on a Calabi-Yau three fold, one obtains an effective 5 dimensional theory [2] which has a structure of two 3 branes situated on the orbifold boundaries. The theory allows $N = 1$ supergravity with gauge and chiral multiplets on the two 3 branes. Thus, it is possible to get phenomenologically interesting $N = 1$ supergravity in four dimensions from the hetrotic string theory. The low energy theory in four dimensions also allows a rich cosmological implications and in recent past some attempts have been made to understand the cosmology of the very early Universe [3].

In this paper we consider a very simple toy model in 5 dimensions and we assume that we reside in one of the two 3 branes which are separated by a distance. In this set-up it has been realized that the effective 4 dimensional cosmology is non-conventional [4]. The Friedmann equation is modified due to the localization of the fields on the brane and also due to the presence of the second brane which constrains the matter energy densities and the equation of states of matter contents of the two branes. The extra 5th dimension is assumed to be orbifold, $y = -y$ and static in our case. The main goal of this paper is to point out some of the interesting implications of the brane cosmology taking place at a energy scale below four dimensional Planck mass and above the nucleosynthesis scale. It is fairly well recognized that the root of most of the nagging problems of presently observed Universe has some relation to the early Universe. We mention here two of them. The present Universe seems to be extremely flat, isotropic and homogeneous. A small inhomogeneity is measured to be one part in 10^5 by COBE satellite, and second startling observation is that the present observable Universe has a small baryon asymmetry which is noted to be roughly one part in 10^{10} measured from the abundances of light elements synthesized at the time of nucleosynthesis. The small inhomogeneity of the Universe can be explained by the quantum fluctuations of the scalar fields either taking part directly in cosmic inflation or sitting in the minimum of their potential but making their presence via quantum fluctuations. On the other hand the observed baryon asymmetry can also be explained quite elegantly in the early Universe because of the presence of preferred time and the expansion of the Universe which leads to out-of-equilibrium decay of massive particles via explicit CP violation interactions. In this paper we will consider one such example of baryogenesis in the supersymmetric theories which is known as Afleck-Dine (AD) mechanism [5]. Strictly speaking we will be treating the branes as hypersurfaces and also assume that initially the branes are supersymmetric and due to some known or unknown reasons supersymmetry is broken at a suitable scale to solve hierarchy problem in our brane. Regarding this we are not realizing to solve the hierarchy problem as proposed in Refs. [6]. However, we remind ourselves that the presence of the extra dimension could play a great role in the whole discussion. So, we move on to describing the basics of the non-conventional cosmology in our brane.

A simple isotropic and homogeneous cosmology can be described by a useful parameter known as the Hubble parameter which explains the rate of expansion of the Universe for a given matter content. It has been noticed that the presence of the extra brane and the condition that the matter content in our own brane is localized within our brane leads to a simple modification to the expansion equation [4].

$$H^2 = \frac{8\pi}{3M_{\rm p}^2}\rho\left[1+\frac{\rho}{2\lambda}\right]+\frac{\Lambda_4}{3}+\frac{\mathcal{E}}{a^4}\,, \qquad (1)$$

where Λ_4 is a four dimensional cosmological constant, ρ is the energy density of the matter stuck to the brane and the last term signifies the influence of bulk gravitons on the brane, which we will neglect at the moment. The brane tension λ relates the four dimensional Planck mass $M_{\rm p} \approx 10^{19}$GeV to the five dimensional Planck scale M_5 via

$$M_{\rm p} = \sqrt{\frac{3}{4\pi}}\left(\frac{M_5^2}{\sqrt{\lambda}}\right) M_5 \,. \tag{2}$$

It is noticeable from Eq. (1) that there is an extra contribution to the right-hand side of the Friedmann equation. If we demand that successful nucleosynthesis occurs then the second term proportional to ρ^2 has to play a negligible role at a scale $\sim \mathcal{O}$(MeV) corresponding to the era of Big Bang nucleosynthesis, thus we have to assume that the modified Friedmann equation paves the usual term in the right-hand side of Eq. (1), which is just linear in energy density ρ. This naturally leads to constraining the brane tension as $\lambda > (1\text{MeV})^4$. This means that the Universe evolves exactly in a familiar fashion even in the presence of branes at energy scales lower than a MeV. However, there could be a significant departure from the usual lore at very high energy scales, especially when $2\lambda < \rho$. In this regime the expansion rate of the Universe is certainly dominated by the ρ^2 term in the right-hand side of Eq. (1). Our aim is to illustrate that perhaps we can accommodate the non-conventional term in Eq. (1) for solving some of the problems, such as excess gravitino production during reheating. The ρ^2 term certainly assist inflation because the Hubble friction term is more dominant in this case. However, the same friction terms leads to slow down the expansion of the Universe during the radiation era. In this context it is worth noticing that the mass splitting between the bosons and the fermions in the supersymmetric theories is ~ 1TeV, if we believe that the hierarchy problem is solved by invoking supersymmetry. The mass scale is also known to be the mass of the gravitino $m_{3/2} \approx M_{\rm susy}^2/M_{\rm p}$. If the mass scale corresponding to the brane tension is assumed to be smaller than the mass of the gravitino, then there lies some interesting possibilities which we will explore in this paper.

Here we write down the energy conservation equation for the matter which is strictly residing within the brane.

$$\dot{\rho} + 3H(\rho + p) = 0 \,. \tag{3}$$

It is noticeable that the conservation equation holds for the matter field living in the brane, however, this would not be the case if there were non-minimal coupling of a scalar field to gravity residing in the bulk and the brane. Interesting cosmology will be discussed elsewhere in this direction, but for the moment we follow Eq. (3). This has an obvious consequence for the scalar field dominating the early Universe during the inflationary phase. It has has been pointed out in Refs. [4,7,8] that inflation is well supported by the ρ^2 contribution and as we had argued earlier the dominance of the friction term possibly leads to many e-foldings of inflation. For our purpose it is the last $50-60$ e-foldings of inflation would be sufficient to form the structure formation in the Universe. The possibility of chaotic inflation with massive inflaton field ($V(\phi) = m^2\phi^2/2$) has been discussed in Ref. [7] with a vanishing four dimensional cosmological constant. Whatsoever be the deep-rooted reason we always assume that the four dimensional cosmological constant is set to be zero from the onset of inflation. The density perturbation produced by the scalar field ϕ during inflation has been compared to that of the COBE result and it has been realized that chaotic inflation can occur for field values below the four dimensional Planck mass $\phi_{\rm cobe} \approx 10^2 M_{\rm p}^{1/3}\lambda^{1/6} < M_{\rm p}$ but above the five dimensional scale M_5. The mass of the inflaton field has also been found to be constrained $m \approx 5\times 10^{-5}M_5$, which essentially translates to $m \approx 10^{-5}M_{\rm p}^{1/3}\lambda^{1/6}$, from Eq. (2). Hence, for $\lambda \approx \mathcal{O}(\text{GeV})^4$, the mass could be $m \sim \mathcal{O}(10)$GeV, and $\phi_{\rm cobe} \sim \mathcal{O}(10^8)$GeV. Thus the scale of inflation is determined by the brane tension and depending on its value inflation could take place at extremely low scale. One of the most important consequence of having inflation at a low scale is the low reheat temperature and various other physical implications which we will describe next.

It is known to us that inflation leads to extremely cold Universe because the entropy generated before and during inflation redshifts away, thus it is necessary to attain thermalization at a scale above the nucleosynthesis scale to preserve the successes of the Big Bang model. In this section we would like to estimate the reheat temperature of the Universe in our scenario. We notice that after the end of inflation the scalar field begins oscillating coherently at the bottom of the potential and for the massive inflaton the average pressure density vanishes during the oscillations, and $\rho_\phi \propto a^{-3}$ where a is the scale factor. If we denote $\rho_{\phi\rm i}$ and $a_{\rm i}$ as the inflaton energy density and the scale factor at the beginning of the coherent oscillations, then the Hubble expansion as a function of a scale factor is given by

$$H^2(a) \approx \frac{8\pi}{3M_{\rm p}^2}\frac{\rho_{\phi\rm i}^2}{2\lambda}\left(\frac{a_{\rm i}}{a}\right)^6 \,. \tag{4}$$

If we assume that reheating of the Universe is very efficient then we can assume that the potential energy stored in inflaton converts to the kinetic energy in a form of radiation. If the decay rate of the inflaton is denoted by Γ_ϕ, then equating $H(a)$ to Γ_ϕ leads to an expression for the scale factors. Now, as we had advertised, if we assume instantaneous reheating then the energy density in radiation $\rho_{\rm r} = \left(\pi^2/30\right) g_* T_{\rm rh}^4$, where g_* is the relativistic degrees of freedom and the reheat temperature $T_{\rm rh}$ is given by

$$T_{\rm rh} \approx \left(\frac{\Gamma_\phi M_{\rm p}\sqrt{\lambda}}{g_*}\right)^{1/4} \,. \tag{5}$$

For the massive boson decay $\Gamma_\phi \approx \alpha m_\phi$, where α is dimensionless Yukawa coupling less than one, and taking the limit on $m_\phi \approx 10^{-5} M_{\rm p}^{1/3}\lambda^{1/6}$ we get

$$T_{\rm rh} \approx \left(10^{-5/4} M_{\rm p}^{1/3}\lambda^{1/6}\right)\left(\frac{\alpha}{g_*}\right)^{1/4}\,, \tag{6}$$

$$\approx 10^{15/4} m_\phi \left(\frac{\alpha}{g_*}\right)^{1/4}\,. \tag{7}$$

For the brane tension $\lambda \sim \mathcal{O}(1)$GeV, reheat temperature could be $T_{\rm rh} \approx \mathcal{O}(10^3)$GeV, assuming $g_* \sim \mathcal{O}(100)$ and $\alpha \approx 0.01$. However, the reheat temperature is always more than the brane tension. This is a direct consequence of inflation occurring at low scales. Inflation at such a scale is desirable from the point of view of nucleosynthesis which we briefly describe here.

If we believe that supersymmetry is needed to solve the hierarchy between the electro-weak scale and the four dimensional Planck mass then the gravitino mass must be no higher than ~ 1 TeV. Since we know that gravitino coupling to matter is Planck mass suppressed, the life time of gravitino at rest is quite long $\tau_{3/2} \sim M_{\rm p}^2/m_{3/2}^3 \sim 10^5\left(m_{3/2}/{\rm TeV}\right)^{-3}$ sec [10]. If the gravitino decays to either gauge bosons and its gaugino partner or if it decays to high energetic photons, synthesis of light elements can be in danger by disturbing the number density of baryon to photon ratio required for the successful nucleosynthesis. However, if the Universe thermalizes at a temperature which is as low as $\mathcal{O}(10^3)$ GeV, the thermal production of gravitinos is also suppressed, but gravitinos could also be produced non-perturbatively during preheating [11], which we do not consider here.

In order to study the gravitino abundance we need to study the Boltzmann equation for the gravitino number density $n_{3/2}$ [9].

$$\frac{dn_{3/2}}{dt} + 3Hn_{3/2} = \langle\Sigma_{\rm tot}v_{\rm rel}\rangle n_{\rm rad}^2 - \frac{m_{3/2}}{\langle E_{3/2}\rangle}\frac{n_{3/2}}{\tau_{3/2}}\,, \tag{8}$$

where $\langle ...\rangle$ represents thermal average, $n_{\rm rad}$ is the number density of relativistic particles $n_{\rm rad} \propto T^3$, $v_{\rm rel}$ is the relative velocity of the scattering radiation which in our case $\langle v_{\rm rel}\rangle = 1$, and the factor $m_{3/2}/\langle E_{3/2}\rangle$ is the average Lorenz factor. We notice that in radiation era the non-conventional brane cosmology gives the following Hubble rate of expansion

$$H \approx \left(\frac{4\pi^5}{3}\right)^{1/2}\frac{g_*}{30}\frac{T^4}{\sqrt{\lambda}M_{\rm p}}\,. \tag{9}$$

In supersymmetric version $g_* \sim 300$, if the reheat temperature is more than the masses of the superpartners. It is worth mentioning that the scale factor during the radiation era follows $a(t) \propto t^{1/4}$, which is contrary to the standard Big Bang scenario where $a(t) \propto t^{1/2}$. However, we must not forget that the derivation is based on the fact that we are in a regime where $\rho > 2\lambda$. In Eq. (8), after the end of inflation the first term in the right-hand-side dominates the second. If we assume the adiabatic expansion of the Universe $a \propto T^{-1}$ then we can rewrite Eq. (8) as $Y_{3/2} = (n_{3/2}/n_{\rm rad})$.

$$\frac{dY_{3/2}}{dT} \approx -\frac{\langle\Sigma_{\rm tot}\rangle n_{\rm rad}}{HT}\,. \tag{10}$$

We notice that we can integrate the temperature dependence from this equation and we mention here that the above expression is exactly the same as in the standard Big Bang case [9]. However, Eq. (10) does not produce the correct value of $Y_{3/2}$, since the true conserved quantity is the entropy per comoving volume, in our case if we assume the gravitinos do not decay within the time frame we are interested in, then we may be able to get the abundance expression at two different temperatures

$$Y_{3/2}(T) \approx \frac{n_{\rm rad}(T_{\rm rh})\langle\Sigma_{\rm tot}\rangle}{H(T_{\rm rh})}\,. \tag{11}$$

Here we assume that the initial abundance of gravitinos at $T_{\rm rh}$ is known to us. The total cross-section $\Sigma_{\rm tot} \propto 1/M_{\rm p}^2$, and $n_{\rm rad}(T_{\rm rh}) \propto T_{\rm rh}^3$, we finally get an expression for the gravitino abundance at temperature T

$$Y_{3/2}(T \ll 1{\rm MeV}) \approx 10^{-2}\frac{\sqrt{\lambda}}{T_{\rm rh}M_{\rm p}}\,. \tag{12}$$

The above expression is an important one and now we are in a position to estimate the abundance for gravitinos. First of all we mention that the abundance equation is in stark contrast to the conventional one $Y_{3/2} \approx 10^{-2}(T_{\rm rh}/M_{\rm p})$, where the reheat temperature appears in the numerator rather than in denominator. If we assume that after their creation during reheating their number density is conserved, then for $T_{\rm rh} \approx 10^3$GeV and $\lambda \approx (1{\rm GeV})^4$, we get an extremely small abundance of gravitinos $Y_{3/2} \approx 10^{-24}$. However, for the similar parameter, the conventional Big Bang cosmology would predict the abundance $Y_{3/2} \approx 10^{-18}$. Thus we find that it is possible to solve the gravitino crisis in the standard Big Bang scenario if we have a brane cosmology. However, we should also study other important aspects of the brane cosmology and to illustrate another example we concentrate upon the issue of baryogenesis.

An important mechanism for generating baryon asymmetry is through the decay of sfermion condensate proposed in Ref. [5], and known as AD mechanism. Let us consider sfermions condensate denoted by ψ and a simple potential for ψ which is lifted by breaking supersymmetry at a suitable scale.

$$V \approx \tilde{m}^2\psi^2\,, \tag{13}$$

where $\tilde{m}$ is related to the supersymmetry breaking scale. A large baryon asymmetry can be generated if there is a baryon number violating operator, such as $\langle A\rangle \neq 0$. The baryon number density stored in the sfermions oscillations is given by [5]

$$n_{\rm B} = \epsilon \left(\frac{\psi_0^2}{M_{\rm G}^2}\right)\frac{\rho_\psi}{\tilde{m}}\,, \tag{14}$$

where ψ_0 is the initial amplitude of the sfermions oscillations, $M_{\rm G}$ can be assumed to be the intermediate scale could be supersymmetric grand unification scale, and $\epsilon(\psi_0^2/M_{\rm G}^2)$ is the net baryon number generated by the decay of ψ. As we know in general that the inflaton begins oscillating when $H \sim m_\phi$ at $a = a_\phi$ and oscillations of the sfermions begin quite late when $H \sim \tilde{m}$ at $a = a_\psi$. However, in our case since the inflationary scale could be quite low depending on the brane tension, the situation could be altered, sfermions can start oscillating much before inflation begins, and thus realization of the AD mechanism could be threatened. Since our calculation is quite general, we are free to choose the brane tension at our will, so we proceed with our calculation. In our non-conventional cosmology if we assume that the inflaton field initially dominates the dynamics then $H \sim (\rho_\phi/M_{\rm p}\sqrt{\lambda})(a_\phi/a)^3$, and for quadratic inflationary potential $H \sim (m_\phi^2\phi^2/M_{\rm p}\sqrt{\lambda})(a_\phi/a)^3$. So, when $H \sim \tilde{m}$, the two scale factors a_ϕ and a_ψ can be related to each other $a = a_\psi = \left(m_\phi^2\phi^2/M_{\rm p}\sqrt{\lambda}\tilde{m}\right)^{1/3} a_\phi$. If inflaton decays to radiation then $\Gamma_\phi = m_\phi^3/M_{\rm p}^2 = H$, where we have assumed that the inflaton decays with a very small coupling, then it is possible to find out the scale factor $a = a_{d\phi} = \left(M_{\rm p}\phi^2/m_\phi\sqrt{\lambda}\right)^{1/3} a_\phi$. Now it is very easy to verify that $a_\psi/a_{d\phi} < 1$ and also $\rho_\phi/\rho_\psi \approx \left(M_{\rm p}\sqrt{\lambda}/\tilde{m}\psi_0^2\right) \gg 1$. This ensures that the Universe is dominated by the inflaton energy provided the amplitude of the sfermions $\psi_0 \ll \left(M_{\rm p}\sqrt{\lambda}/\tilde{m}\right)^{1/2}$. If we assume efficient reheating then we can estimate the energy density in the radiation, which is given by

$$\rho_{{\rm r}\phi} = \left(\frac{m_\phi^{5/3}M_{\rm p}^{1/3}\phi^{8/3}}{\lambda^{1/6}}\right)\left(\frac{a_\phi}{a}\right)^4 . \tag{15}$$

and corresponding Hubble parameter is given by $H = \left(m_\phi^{5/3}\phi^{8/3}/M_{\rm p}^{2/3}\lambda^{2/3}\right)(a_\phi/a)^4$. Since the sfermions decay after inflaton has decayed and reached its thermalization it is important to know the interaction rate of inflaton which leads to thermalization. Following the argument in Refs. [12,13] thermalization of the inflaton decay products can be estimated by

$$\begin{aligned}\Gamma_{\rm T} &\sim n_\phi\sigma \sim m_\phi\phi^2\left(\frac{a_\phi}{a}\right)^3\left(\frac{\alpha}{m_\phi}\right)^2\left(\frac{a}{a_{{\rm d}\phi}}\right)^2 ,\\ &\sim \left(\frac{\alpha^2\phi^{2/3}\lambda^{1/3}}{m_\phi^{1/3}M_{\rm p}^{2/3}}\right)\left(\frac{a_\phi}{a}\right) ,\end{aligned} \tag{16}$$

where α is a gauge coupling constant, σ is the elastic scattering cross-section. Thermalization occurs when $a_{\rm T} \sim \left(m_\phi^2\phi^2\lambda^{-1}/\alpha^2\right)^{1/3} a_\phi$. Upon thermalization the final baryon to entropy ratio can be estimated with the help of Eqs. (14) and (15). The entropy density is given by $s = (\rho_{{\rm r}\phi}(a_{\rm T}))^{3/4} \sim \left(m_{\rm p}^{1/4}\lambda^{7/8}/m_\phi^{3/4}\right)\alpha^2$ and the baryon to entropy ratio is given by

$$\frac{n_{\rm B}}{s} = \left(\frac{\epsilon\psi_0^4 m_\phi^{3/4}}{M_{\rm G}^2 M_{\rm p}^{5/4}\lambda^{3/8}}\right) . \tag{17}$$

It is worth mentioning here that the final expression for the baryon asymmetry is independent of the mass of sfermions, which is in contrast to the result obtained in standard cosmology [13]. However, the ratio depends on the brane tension. For $\lambda \sim (1{\rm GeV})^4$, $\epsilon \sim 10^{-2}$, $M_{\rm G} \sim 10^{15}$ GeV, $m_\phi \sim 100$ GeV, to obtain the desired ratio $n_{\rm B}/s \sim 10^{-10}$, the amplitude of the $\psi_0 \leq 10^{10}$GeV. Clearly such a possibility is not ruled out in the non-conventional brane cosmology. This could be an example of AD baryogenesis at extremely low scale. At higher inflationary scale it is rather easier to realize this situation, but that would also require higher brane tension. In this paper we have raised two issues relevance to the early Universe, we have shown that it is possible to yield a small abundance of gravitinos produced in a thermal bath in our set-up and it is also possible to obtain a low scale AD baryogenesis. It would be interesting to consider other possibilities and work in this direction is in progress. ACKNOWLEDGEMENTS: The author is supported by the INLAKS foundation.

[1] P. Horava and E. Witten, Nucl. Phys. B **460** (1996) 506; Nucl. Phys. B **475** (1996) 94.

[2] A. Lukas, B. A. Ovrut, K. Stelle and D. Waldram, Phys. Rev. D **60** (1999) 086001; J. E. Ellis, Z. Lalak, S. Pokorski, W. Pokorski, Nucl. Phys. B **540** (1999) 149.

[3] A. Lukas, B. A. Ovrut, D. Waldram, Phys. Rev. D **61** (2000) 023506.

[4] P. Binetruy, C. Daffayet and D. Langlois, Nucl. Phys. B **565** (2000) 269; P. Binetruy, C. daffayet, U. Ellwanger and D. langlois, Phys. Lett. B **477** (2000) 269; T. Shiromizu, K. Maeda and M. Sasaki, gr-qc/9910076.

[5] I. Affleck and M. Dine, Nucl. Phys. B **249** (1985) 361.

[6] N. Arkani-Hamed, S. Dimopoulos, G. Dvali, Phys. Lett. B **249** (1998) 263; L. Randall and R. Sundrm, Phys. Rev. Lett. **83** (1999) 3370.

[7] R. Maartens, D. Wands, B. A. Bassett and I. P. C. Heard, Phys. Rev. D **62** (2000) 041301.

[8] E. J. Copeland, A. R. Liddle and J. E. Lidsey, astro-ph/0006421.

[9] E. W. Kolb and M. Turner, *The Early Universe*, Addison-Wesley, 1993; T. Moroi, P.hD. Thesis, hep-ph/9503210.

[10] J. Cline and S. Raby, Phys. Rev. D **43** (1991) 1781.

[11] A. L. Maroto and A. Mazumdar, Phys. Rev. Lett. **84** (2000) 1655; R. Kallosh, L. Kofman, A. Linde and A. V. Proyen, Phys. Rev. D **61** (2000) 103503; G. F. Giudice, A. Riotto and I. Tkachev, JHEP 9908 (1999) 009.

[12] J. Ellis, D. V. Nanopoulos and K. A. Olive, Phys. Lett. B **184**, (1987) 37.

[13] J. Ellis, K. Enqvist, D. V. Nanopoulos and K. A. Olive, Phys. Lett. B **191**, (1987) 343.